Exercise 44

Suppose that $f(4) = 2$, $g(4) = 5$, $f'(4) = 6$, and $g'(4) = -3$. Find $h'(4)$.

(a) $h(x) = 3f(x) + 8g(x)$ (b) $h(x) = f(x)g(x)$

(c) $h(x) = \dfrac{f(x)}{g(x)}$ (d) $h(x) = \dfrac{g(x)}{f(x) + g(x)}$

Solution

Calculate the derivatives using the product and quotient rules.

Part (a)

Differentiate $h(x)$.

$$h'(x) = \frac{d}{dx}[3f(x) + 8g(x)] = 3f'(x) + 8g'(x)$$

Evaluate it at $x = 4$.

$$h'(4) = 3f'(4) + 8g'(4) = 3(6) + 8(-3) = -6$$

Part (b)

Differentiate $h(x)$.

$$h'(x) = \frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Evaluate it at $x = 4$.

$$h'(4) = f'(4)g(4) + f(4)g'(4) = (6)(5) + (2)(-3) = 24$$

Part (c)

Differentiate $h(x)$.

$$h'(x) = \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

Evaluate it at $x = 4$.

$$h'(4) = \frac{f'(4)g(4) - g'(4)f(4)}{[g(4)]^2} = \frac{(6)(5) - (-3)(2)}{(5)^2} = \frac{36}{25}$$

Part (d)

Differentiate $h(x)$.

$$h'(x) = \frac{d}{dx}\left[\frac{g(x)}{f(x) + g(x)}\right] = \frac{g'(x)[f(x) + g(x)] - [f(x) + g(x)]'g(x)}{[f(x) + g(x)]^2}$$

Evaluate it at $x = 4$.

$$h'(4) = \frac{g'(4)[f(4) + g(4)] - [f'(4) + g'(4)]g(4)}{[f(4) + g(4)]^2} = \frac{(-3)[(2) + (5)] - [(6) + (-3)](5)}{[(2) + (5)]^2} = -\frac{36}{49}$$